## FACULTY OF HEALTH ПAПIBIA UПIVERSITY AND APPLIED SCIENCES OF SCIEПCE AПD TECHПOLOGY DEPARTMENT OF MATHEMATICS AND STATISTICS

| QUALIFICATION: Bachelor of science; Bachelor of science in Applied Mathematics and Statistics |  |
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| QUALIFICATION CODE: 07BSOC; 07BAMS | LEVEL: 5 |
| COURSE CODE: CLS502S | COURSE NAME: CALCULUS 1 |
| SESSION: JANUARY 2020 | PAPER: THEORY |
| DURATION: 3 HOURS | MARKS: 100 |


| SECOND OPPORTUNITY EXAMINATION QUESTION PAPER |  |
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| EXAMINER | Dr N. Chere and Mrs Y.Shaanika-Nkalle |
| MODERATOR: | Prof Gunter Heimbeck |


| INSTRUCTIONS |
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| 1. Answer ALL the questions in the booklet provided. <br> 2. Show clearly all the steps used in the calculations. <br> 3. All written work must be done in blue or black ink and sketches must <br> be done in pencil. |

## PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

## SECTION A: [Short answer questions] [ $2 \frac{1}{2}$ marks for each question]

## QUESTION 1 [25]

1.1. Suppose that $\lim _{x \rightarrow-2} f(x)=12, \lim _{x \rightarrow-2} g(x)=-3$. Then find
1.1.1. $\lim _{x \rightarrow-2}(\sqrt{3 f(x)}+g(x))=$
1.1.2. $\lim _{x \rightarrow-2}\left((g(x))^{2}+x\right)$
1.1.3. $\lim _{x \rightarrow-2}\left(\frac{x^{2}+x g(x)}{f(x)}\right)=$
1.1.4. $\lim _{x \rightarrow-2}\left(2 x+(f(x))^{2}\right)=$
1.2. Determine the following derivatives.
1.2.1. $\frac{\mathrm{d}}{\mathrm{dx}}\left(\sin \left(\frac{1}{\mathrm{x}}\right)\right)=$
1.2.2. $\frac{d}{d x}\left(e^{\cos x}\right)=$
1.2.3. If $y=\ln (\sin x)$, then $\left.\frac{d y}{d x}\right|_{x=\frac{\pi}{4}}=$
1.3. Suppose that $f$ and $g$ are continuous functions such that $g(4)=2$

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\text { and } \lim _{x \rightarrow 4}(2 f(x)+3 g(x))=20 . \text { Then the value of } f(4)=
$$

1.4. The domain of the function $f(x)=\sqrt{4-9 \mathrm{x}^{2}}$ is equal to
1.5. Suppose a function $f$ has the property that for all real numbers $x, 1-x^{2} \leq f(x) \leq \cos x$. Then $\lim _{x \rightarrow 0} f(x)=$

## SECTION B [Workout Problems]

## QUESTION 2 [75]

2.1. Let $f(x)=\sqrt{2 x+2}$. Then
2.1.1. find a formula for $f^{-1}(x)$.
2.1.2. state the range of $f^{-1}$.
2.2. Evaluate the following limits if it exists.
2.2.1. $\lim _{x \rightarrow-\infty} \frac{2 x^{4}+4 x^{2}+3}{x^{2}+2 x^{3}+1}$
2.2.2. $\lim _{x \rightarrow-1} \frac{\ln \left(x^{3}+2\right)}{x+1}$
2.2.3. $\lim _{x \rightarrow 2} \frac{\sqrt{2 x+4}-\sqrt{8}}{x-2}$
2.2.4. $\lim _{x \rightarrow 2} \frac{x^{2}-2 x}{x^{3}-8}$
2.3. Let $f(x)=x^{2}-x$. Find $f^{\prime}(x)$ by using the limit definition of derivative.
2.4. Use the precise definition of limit to prove that $\lim _{x \rightarrow 3}(2 x+3)=9$.
2.5. Use chain rule to find $\frac{d y}{d x}$ if $y=\sin (\ln 2 x)$
2.6. If $x+y=2 x y^{2}$ Then
2.6.1. Use implicit differentiation to solve and express $\frac{d y}{d x}$ in terms of x and y .
2.6.2. Use the result in (2.6.1) to find an equation of a tangent line to the curve $x+y=2 x y^{2}$ at $(-1,-1)$.
2.7. Suppose $f(x)=-2 x^{3}-x+3$. Then
2.7.1. find $\left(\mathrm{f}^{-1}\right)^{\prime}(\mathrm{x})$
2.7.2. use (3.6.2) to find $\left(f^{-1}\right)^{\prime}(0)$
2.8. Let $f(x)=2 x^{3}-3 x^{2}-12 x$.
2.8.1. find the local maximum and local minimum value of $f$ if there are any.
2.8.2. the intervals on which $f$ is increasing and when where it is decreasing.
2.8.3. the open intervals on which the graph of $f$ is concave upward and on which the graph of $f$ is concave down ward.
2.8.4. the inflection point(s).

